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# TEACHING STATEMENT

## 2.1 Scheduled Teaching Activities at UCI

During my years at UCI, I have taught 37 different regularly scheduled courses. These include 26 lower-division classes and 11 lower-division classes:
MATH 2A – *Differential Calculus* (F'10 (2 sections), W'11, S'11, F'11, W'13, S'14);
MATH 2B – *Integral Calculus* (W'08, S'09, F'10);
MATH 2D – *Multivariable Calculus* (S'12);
MATH 2J – *Infinite Series and Linear Algebra* (S'10)*;*
MATH 3A – *Linear Algebra* (S'10), *MATH 4 – Math for Economists* (W'11);
MATH 6B – *Boolean Algebra & Logic* (W'08),*;*
MATH 13 – *Introduction to Abstract Mathematics* (F'11, W'12, S'12, F'12 (2 sections), W'13, F'13,
W'14, S'14, F'14. W'15);
MATH 120A – *Introduction to Group Theory* (W'12, F'12, F'13, F'14, W'14),
MATH 120B – *Introduction to Ring Theory* (S'09, S'13, W'14, W'15, S'15)
MATH 120C – *Introduction to Galois Theory* (S'13).

## 2.2 Development of innovative teaching strategies

Students' perception of effective teaching is one of the strongest predictor for re-enrollment; in this sense, good teaching is an extremely powerful retention tool. I set very high expectations for my students, but at the same time I strive to implement good teaching practices (well-organized lectures, clear explanations, helpful office hours, group study sessions, among others) and I keep experimenting with new strategies to promote active learning.
During the past few years I had a chance to teach several courses at UCI, with enrollment varying from 10 to 240, on a wide range of topics (from differential calculus to Galois Theory). I have coordinated large sessions of calculus and helped develop Webwork (an online homework system). Teaching has always given me great joy and satisfaction. The following student comment is indicative of my teaching style.
'She doesn't always tell us the answer; she makes us think. Her teaching style is interactive; people answer her questions and they are allowed to do problems on the whiteboard, which allows them to learn better.'
I have received teaching awards from Princeton University (2002), Cornell University (2006) and UCI (2014). My high teaching evaluations are the result of a conscious attempt to shift the goal of my lectures from delivering instruction to producing learning. I am continuously experimenting with new pedagogical techniques aimed at creating a collaborative environment in which every student takes ownership of his/her own learning. Some of these strategies are described below; details about implementation are available in a teaching statement on my website.

*(a) Guide students to the discovery of and appreciation for the true meaning of mathematics*

- Lead students to invent solutions, not just memorize procedures.
  - Example from a Linear Algebra course (Math 3A): Provide the augmented matrix for three linear systems with more equations than variables, exhibiting 0, 1, or infinitely many solutions, respectively.
- Lead students to explore patterns, not just memorize formulas.
- Lead students to formulate conjectures, not just do exercises.
  - Example from an Introduction to Proofs course (Math 13): Evaluate the Phi Euler function $\varphi(n)$ for all $n = 1, \ldots, 16$. What is the biggest value that $\varphi(n)$ can take? (Later, students can be asked to prove that $n$ is prime if and only if $\varphi(n)$ is prime.)
- Lead students to reverse-engineer some of the proofs presented in abstract algebra courses

and discover how the key definitions in the course were created.

- Example from a Ring Theory course (Math 120B): Through a class discussion, help students rediscover the definition of a prime ideal by investigating what properties should an ideal satisfy if we want the quotient ring to be an integral domain.

*(b) Invite the students to think mathematically, and think creatively*

- Replace standard assignments with *'Prove or Disprove'* problems.
  - Example from a Linear Algebra course (Math 3A): Prove or disprove: (i) Every 5 vectors in $\mathbb{R}^3$ are linearly dependent. (ii) Every 3 vectors in $\mathbb{R}^5$ are linearly independent.
  - Example from a Group Theory course (Math 120A): Prove or disprove: (i) The set $\{f \in C^1(\mathbb{R})$ such that $f'(0) = 3\}$ is a subgroup of $C^1(\mathbb{R}, +)$. (ii) The set $\{f \in C^1(\mathbb{R})$ such that $f'(3) = 0\}$ is a subgroup of $C^1(\mathbb{R}, +)$.
- Assign a Conjecture Project at the end of the course.
  - Example from an Introduction to Proofs course (Math 13): Make a conjecture about the total number of squares in a $n\times n$ chessboard, then prove it.
- Invite the students to complete a theorem statement, then prove it.
  - Example from a Linear Algebra course (Math 3A): The system Ax=b is always consistent if and only if the row echelon form of A has one pivot per ______________ (row or column?).
- Assign creative poofs as final project for a course.
  - Example from a Group Theory course (Math 120A): Use Lagrange's theorem to prove that n!m! divides (n+m)!.

*(c) Engage students in mathematical conversations*

- Plan special group activities for the first day of classes, to encourage active learning and group work, and set up the right tone for the course.
  - The following activity was waiting for students on their desks as they walked in my (Math 3A) Linear Algebra class (on the very first day of classes): `If possible, find constants $a, b, c$ so that the system of equations $2x + 4y = 6, ax + by = c$ has no solutions, 1 solutions, 2 solutions or infinitely many.' Hint: Think geometrically!
- Design worksheets and other think-pair-share activities to do in the classroom.
- Give students time to brainstorm the solution to a problem (typically in small groups), then invite them to come to the board and present their answer.
  - I do this for all my classes with at most 50 students, especially for problems involving several parts. To save time and make the process less intimidating for students, I bring several markers to class and invite different pairs of students to write up the solutions to different parts of the problem *simultaneously* at the board. (For example, if the task was to prove that a given subset is a subgroup, I would split the board in 3 parts and have 3 pairs of students working simultaneously at the board, proving closure, existence of identity element and existence of inverses.) After all the solutions are written up, I ask the students to sit down, and present their answers to the class (correcting mistakes if necessary, adding those quantifiers they probably forgot, including missing justification for certain steps or giving advice on how to make the proof more elegant) and recap. Students get pride in their work, and I have an opportunity to show them what I expect an optimal solution to look like.
- Invite students to use the message board on EEE to share mathematical ideas and answer each other's questions.
- Replace at least one of the office hours with a study session.
  - This is immensely effective for courses with less than 50 students, and much more productive than an office hour. I book a seminar room and invite students to sit in groups and work on problems I prepared, while I go around tables and help various group as they get stuck on a problem. Students learn a lot from the extra practice and from the cooperative learning experiences. Benefits transfer to the lecture as well: Because students get to know each other, they are more at easy working together in class or presenting in front of the room, and less afraid of making mistakes while they answer questions. Most importantly, I see first hand where students are having difficulties, and can address the problem in my lectures. Finally, education literature shows that cooperative learning is particular beneficial for women and under-represented minorities. I believe study sessions are a powerful retention tool.
- Assign two kinds of homework: individual homework, containing standard problems to practice techniques and definitions, and group homework containing harder and more thought-provoking assignments.

- ➢ Here is an example of a problem suitable for cooperative homework for a Group Theory course (Math 120A): Consider the power set of a set X, with the operations of intersection, union, and symmetric difference. In each case, decide whether the binary structure has an identity element and, if so, find the units.

- Use technology to enhance students' involvement in a classroom of 240 students.
  - ➢ Example from a Differential Calculus course (Math 2A): In a mega calculus class, I have often used the dual projector setting, so that I can simultaneously use my laptop (to project i-clicker questions, long statements of word problems, differentiation rules or theorems) and a document camera (to provide step-by-step solutions to the i-clicker questions or the word problems displayed on the other screen, and to give examples, proofs or applications of theorems or rules posted on the slides).
- Summarize the idea of the proof after the solution to a problem is presented.
- Require the completion of a *Proof/Problem Portfolio* at the end of a course, where students present (and reflect upon) the various topics encountered in the class.
  - ➢ Example from an Introduction to Proofs course (Math 13): Students submit a proof portfolio at the end of the quarter. They select some proofs from their homework that are representative of the content of the course, they put their best efforts rewriting them, and they accompany each proof with a reflection. In the commentary, they explain why they are proud of their solution, what difficulties they encountered when they first attempted the proof, and why that particular proof was relevant in the course. I have used a similar portfolio in a Group Theory class.
- Assign reflective tasks at the end of a course.
  - ➢ Example from a Galois Theory course (Math 120C): As summative assessment at the end of the course, ask students to draw a flow chart to illustrate the various steps in the determination of the splitting field of a polynomial (or another Galois Theory concept of their choice). In addition, ask students to write a letter to an imaginary friend in the prerequisite course (ring theory), summarizing the content of the course and highlighting the cool aspects of the theory.

*(d) Create a supportive environment inside and outside the classroom*

- Have an open door policy in the first week of classes, when students are welcome to stop by the office at any time and introduce themselves (submitting, in person, their mathematical biography and a brief survey about themselves).
- Convince students to attend office hours and study sessions, right from the beginning. (Attendance from girls seems to increase if encouraged to come in groups.)
- Follow up with students who appear to be falling behind, or who unexpectedly miss an assignment.
  - ➢ After one such follow-up, one surprised student answered: 'Oh, well thank you. That's refreshing. And I mean that. I've been dragging myself through UCI for the last 2 years and you're the only professor who's ever reached out and expressed anything like that. This is my last quarter here, but thank you for caring. PS: Can you believe I used to be a math major!'
- Alert students of their need to review basic topics or skills by means of an online, electronically graded, not for credit, 'Prerequisites Test' at the beginning of the course. Similarly, a similar 'review test' before exams can inform students about how well they are grasping different portion of the material in preparation for a midterm or a final.
  - ➢ Example from an Integral Calculus (Math 2B): A 'Prerequisites Test' for students entering Math 2B can make them aware of how poorly they remember limits, derivatives, inverse trigonometric functions, and basic trig identities for right triangles.
  - ➢ Example from an Introduction to Proofs (Math 13): A series of tailored online 'review tests' targeting specific skills that are known to be difficult for students (e.g., negating compound quantified statements, rephrasing 'only if' statements in standard 'if… then…' form, properly stating the inductive hypothesis in an induction proof).

*(e) Strengthen students' interest in mathematics, and ease their transition to upper division courses*

- Over the past year I have devoted considerable efforts to redesign our Math 13 – 'Introduction to proofs' course, a transition course that sets the foundations for students' success in upper division math. I have taught the class several times and coauthored a set of notes that ended up replacing the textbook for the class. Beyond covering the basics of logic and illustrating various proof techniques, the notes introduce students to the spirit of mathematics and the methods of mathematical thinking, conveying a sense of appreciation and ownership of mathematical ideas.

- Train the students to pay attention to definitions, and encourage them to read the textbook.
  - Over the year, I have tried several approaches, not all effective. At first, I simply asked students to read the textbook prior to each lecture, write down the relevant definitions on a flash card, and answer a definition question in each quiz. After a while, I realized that it would much more useful to ask students about the concept image associated to each definition, as revealed by the examples and non-examples generated by the students.

    Another very technique that worked very well for me has been 'hiding' new definitions in the homework problems I write for a course, so that students fight with the new concepts on their own before encountering them in lecture. For example, in the very first homework for a group theory class, I would ask students to explore the notions of identity element and units within (concrete examples of) binary structures, before introducing the notion of a group.

- At the start of an introductory upper division course (e.g., first quarter of abstract algebra), help students understand proofs by explaining the thought process behind a neat proof, constantly inviting the students to suggest the next step (or at least explain it). After a few weeks, expect the students to have a higher degree of independence in writing their proofs.

*(f) Always question and reinvent yourself as a teacher*

- Over the past few years, I have dedicated much attention to professional training, seeking to learn the latest concepts in math education and pedagogy. With every class I teach, I tailor my teaching skills to the new set of students, and I learn something new. This never ending search for optimal instructional strategies is one of the most exciting aspects of my job, and is gradually shifting my research interests from abstract algebra and representation theory towards pedagogy and higher education.